11.14.22

LECTURE 36

f_x is the partial derivative wrt x, and we treat y as a constant, + vice versa

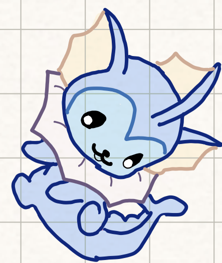
f

$f_x \qquad f_y$

$(f_x)_x \quad (f_x)_y \quad (f_y)_x \quad (f_y)_y$

$\vdots \quad \vdots \quad \vdots \quad \vdots$

$\frac{\partial}{\partial x}$ is partial wrt x



EXAMPLE

$f(x,y) = x^2y - 6xy^3 + 2x - 3y$. Compute f_{xx}, f_{xy}, f_{yx}, f_{yy}

$f_{xx} = f_x(2yx - 6y^3 + 2) = 2y$

$f_{xy} = f_y(2yx - 6y^3 + 2)$
$= 2x - 18y^2$

$f_{yx} = f_x(x^2 - 18xy^2 - 3) = 2x - 18y^2$

$f_{yy} = f_y(x^2 - 18xy^2 - 3)$
$= -36xy$

THEOREM

If f is a function of two variables and the partials are continuous, then:

$$f_{xy} = f_{yx}$$

<u>Corollary</u>: All partial derivatives of a certain order "n" are equal

↳ However, must be the same "number" of x's and y's. i.e. $f_{xxy} = f_{yxx} \neq f_{xxx}$

LECTURE 36

Lecture 36 Problems

1) $u_t = \frac{\partial}{\partial t}(\sin c\omega t \ \sin \omega x) = c\omega \cos(c\omega t) \sin \omega x$

$u_x = \frac{\partial}{\partial x}(\sin c\omega t \ \sin \omega x) = \omega \sin c\omega t \ \cos \omega x$

$u_{xx} = \frac{\partial}{\partial x}(\omega \sin c\omega t \ \cos \omega x) = -\omega^2 \sin \omega x \sin(c\omega t)$

$\boxed{B}$

$u_{tt} = \frac{\partial}{\partial t}(c\omega \cos(c\omega t) \sin \omega x) = -c^2 \omega^2 \sin(\omega x) \sin(c\omega t)$

2)

b. $u_x = \frac{\partial}{\partial x} e^{x+y} \sin(x-y) = e^{x+y} \sin(x-y) + e^{x+y} \cos(x-y)$

$u_{xx} = \frac{\partial}{\partial x}(e^{x+y}(\sin(x-y) + \cos(x-y))) = e^{x+y}[(\sin(x-y) + \cos(x-y)) + \cos(x-y) - \sin(x-y)]$

$u_{yy} = \frac{\partial}{\partial y}(e^{x+y}(\sin(x-y) + \cos(x-y)))$

$\boxed{D}$

3)

1. All same - 3
2. Two same one diff - $3+3$ f_{xxy} f_{yxz} 10 $\boxed{C}$
3. All different - 1